for the case of a uniform pressure distributed over the whole middle surface resting on the outline Γ. If the kinematic boundary conditions on Γ are such that the contour integral in (4.9) vanishes, then the elementary work of the load (4.7) is a total variation of the functional.

REFERENCES

1. Lur'e, A. I., Theory of Elasticity. "Nauka", Moscow, 1970.
2. Lur'e, A. I., Bifurcation of equilibrium of a perfectly elastic body. PMM Vol. 30, № 4, 1966.
3. Zubov, L. M., Variational principles of the nonlinear theory of elasticity. Case of superposition of a small deformation on a finite deformation. PMM Vol. 35, № 5, 1971.
4. Lur'e, A. I., On differentiation of tensors with respect to time. Trudy Leningrad Politechnic Institute. 1971.
5. Appel, P., Theoretical Mechanics, Vol. 1. Fizmatgiz, Moscow, 1960.
6. Kagan, V. F., Principles of the Theory of Surfaces in a Tensor Exposition, Vol. 1. Gostekhizdat, Moscow-Leningrad, 1947.
7. Novozhilov, V. V., Theory of Thin Shells. Sudpromgiz, Leningrad, 1951.
8. Timoshenko, S. P., Stability of Elastic Systems. Gostekhizdat, Moscow, 1955.

Translated by M. D. F.

UDC 539.3

ASYMPTOTIC ANALYSIS OF THE STATE OF STRESS OF AN INFINITE CIRCULAR CYLINDRICAL SHELL

PMM Vol. 40, № 1, 1976, pp. 96-103
D. I. SHERMAN
(Moscow)
(Received July 7, 1975)

The transverse sections of a shell are deformed identically under the effect of external forces which do not vary along the generator. In this case it is admissible to limit oneself to a study of the state of stress of an elastic concentric ring. A large quantity of papers is devoted to this classical problem. Primarily the case of nonthin shells is treated. The exception is in the papers of Ustinov [1, 2], where the state of stress of a very thin ring subjected to normal forces is considered. The stress field in a thin ring, seemingly subjected to both normal and tangential external forces, is also analyzed in this paper by another method.

1. Let S designate a domain occupied by a concentric ring, and L_2 and L_1 its outer and inner bounding circles, respectively. We take the boundary conditions for the first fundamental problem in the usual form

$$\varphi_1(t) + \overline{t\varphi_1'(t)} + \overline{\psi_1(t)} = f_2(t) \text{ on } L_2 \tag{1.1}$$

$$\varphi_1(t) + \overline{t\varphi_1'(t)} + \overline{\psi_1(t)} = f_1(t) + C_1 \text{ on } L_1 \tag{1.2}$$

where $\varphi_1(z)$ and $\psi_1(z)$ are the required functions, regular in the domain S, and $f_1(t)$ and $f_2(t)$ are some functions given on the corresponding curves L_1 and L_2. Examination of this problem from the aspect of the method recommended earlier [3] results in a Fredholm integral equation for the auxiliary density, introduced by observation on the inner or outer boundary of the ring.

Let us indicate schematically the process of deriving the equation. Writing condition (1.1) on the circle L_2 as

$$\varphi_1(t)+\overline{\chi_1(t)}=f_2(t),\quad \chi_1(z)=\frac{R_2^2}{z}\varphi_1'(z)+\psi_1(z) \tag{1.3}$$

we introduce the auxiliary function $\omega(t)$ on L_2 according to the equality

$$\varphi_1(t)-\overline{\chi_1(t)}=2\omega(t). \tag{1.4}$$

A regular function in the ring S

$$\varphi(z)=\varphi_1(z)-\frac{1}{2\pi i}\int_{L_2}\frac{\omega(t)+{}^1/_2 f_2(t)}{t-z}\,dt \tag{1.5}$$

is analytically continuable outside the contour L_2; it is regular everywhere outside L_1 and vanishes at infinity. The equality

$$\varphi(z)=-\frac{1}{2\pi i}\int_{L_1}\frac{\omega(t)+{}^1/_2 f_2(t)}{t-z}\,dt \tag{1.6}$$

is valid outside the contour L_2 We analogously conclude that the function

$$\chi(z)=\chi_1(z)-\frac{1}{2\pi i}\int_{L_2}\frac{\overline{-\omega(t)+{}^1/_2 f_2(t)}}{t-z}\,dt \tag{1.7}$$

is continuable outside the contour L_2 and is given outside this contour by the formula

$$\chi(z)=-\frac{1}{2\pi i}\int_{L_2}\frac{\overline{-\omega(t)+{}^1/_2 f_2(t)}}{t-z}\,dt \tag{1.8}$$

Converting condition (1.2) into

$$\varphi_1(t)-\frac{R_1^2-R_2^2}{\bar t}\overline{\varphi_1'(t)}+\overline{\chi_1(t)}=f_1(t)+C_1\quad\text{on}\quad L_1$$

and taking into account (1.6) — (1.8), we have on L_1

$$\varphi(t_0)+\frac{R_1^2-R_2^2}{\bar t_0}\overline{\varphi'(t_0)}+\overline{\chi(t_0)}=f_1(t_0)+C_1-$$
$$\frac{1}{2\pi i}\int_{L_2}\frac{\omega(t)+{}^1/_2 f_2(t)}{t-t_0}\,dt+\frac{R_1^2-R_2^2}{\bar t_0}\frac{1}{2\pi i}\int\frac{\overline{\omega'(t)+{}^1/_2 f_2'(t)}}{\bar t-\bar t_0}\,\overline{dt}-$$
$$\frac{1}{2\pi i}\int_{L_2}\frac{\omega(t)-{}^1/_2 f_2(t)}{\bar t-\bar t_0}\,\overline{dt}$$

Hence, after simple computations we find the functions $\varphi(z)$ and $\psi(z)$. In order to write them in more compact form, let us introduce the following quantities (which shall henceforth be used):

$$P(z)=P_1(z)+P_2(z),\quad P_2(z)=P_2^{(1)}(z)+P_2^{(2)}(z)$$
$$Q(z)=Q_1(z)+Q_2(z),\quad Q_2(z)+Q_2^{(1)}(z)+Q_2^{(2)}(z)$$
$$G(z)=G^{(1)}(z)+G^{(2)}(z),\quad T(z)=T^{(1)}(z)+T^{(2)}(z)$$

The quantities $P(z)$ and $Q(z)$ are regular outside L_1 and are given there by

$$P_1(z) = \frac{1}{2\pi i}\int_{L_1}\frac{f_1(t)}{t-z}dt, \quad Q(z) = \frac{1}{2\pi i}\int_{L_1}\frac{\overline{f_1(t)}}{t-z}dt, \quad \lambda = \frac{R_1^2}{R_2^2} \tag{1.9}$$

$$P_2^{(1)}(z) = \frac{1}{4\pi i}\int_{L_2}\frac{f_2(t)\,dt}{t-z/\lambda}, \quad P_2^{(2)}(z) = -\frac{1-\lambda}{4\pi i}\int_{L_2}\frac{t\overline{f_2'(t)}}{t-z/\lambda}dt$$

$$Q_2^{(1)}(z) = \frac{1}{4\pi i}\int_{L_2}\frac{\overline{f_2(t)}}{t-z/\lambda}dt$$

$$Q_2^{(2)}(z) = -\frac{R_1^2-R_2^2}{z}\frac{1}{4\pi i}\int_{L_2}\frac{f_2'(t)}{}dt$$

The remaining $G(z)$ and $T(z)$ depend on the unknown $\omega(t)$, where their components are

$$G^{(1)}(z) = -\frac{1}{2\pi i}\int_{L_2}\frac{\omega(t)}{t-z/\lambda}dt, \quad G^{(2)}(z) = -\frac{1-\lambda}{2\pi i}\int_{L_2}\frac{t\overline{\omega'(t)}}{t-z/\lambda}dt \tag{1.10}$$

$$T_{(z)}^{(1)} = \frac{1}{2\pi i}\int_{L_2}\frac{\overline{\omega(t)}}{t-z/\lambda}dt$$

$$T^{(2)}(z) = -\frac{R_1^2-R_2^2}{z}\frac{1}{2\pi i}\int_{L_2}\frac{\omega'(t)}{t}dt$$

Using the relationships (1.9) and (1.10), we arrive at the equalities

$$\varphi(z) = P(z) + G(z), \quad \frac{R_1^2-R_2^2}{z}\varphi'(z) + \chi(z) = Q(z) + T(z)$$

from which we find for the function

$$\chi(z) = [Q(z) + T(z)] - \frac{R_1^2-R_2^2}{z}[P'(z) + G'(z)]$$

Now passing over to the equality (1.4) on the contour L_2 and taking into account (1.6) and (1.7), we obtain the following integral equation for the density $\omega(t)$:

$$\omega(t_0) = \varphi(t_0) - \overline{\chi(t_0)} + \frac{1}{2\pi i}\int_{L_2}\frac{\omega(t)}{t}dt + \frac{1}{4\pi i}\int_{L_2}f_2(t)\left[\frac{dt}{t-t_0} + \frac{\overline{dt}}{\bar t-\bar t_0}\right]$$

Having performed the computations outlined, we obtain the following integral equation for $\omega(t)$:

$$\omega(t_0) = G(t_0) - T(t_0) - (1-\lambda)\,t_0\overline{G'(t_0)} + \frac{1}{2\pi i}\int_{L_2}\frac{\omega(t)}{t}dt + R(t_0) \tag{1.11}$$

where $R(t_0)$ is a known function given by the equality

$$R(t_0) = P(t_0) - \overline{Q(t_0)} - (1-\lambda)\,t_0\overline{P'(t_0)} + \frac{1}{4\pi i}\int_{L_2}f_2(t)\left[\frac{dt}{t-t_0} + \frac{\overline{dt}}{\bar t-\bar t_0}\right]$$

Certain integrals in the right side of (1.11) contain derivatives with respect to the density $\omega(t)$ by performing integration by parts therein, we finally arrive at a Fredholm equation in $w(t)$.

The density $\omega(t)$ contained in the integrals of the Fredholm equation (1.11) is a continuous complex function of the complex variable t. Meanwhile the parameter characterizing the closeness of the ring boundaries is present only in the kernel of the integral equation (1.11). It is difficult to remove this parameter directly outside the integral sign. However, this is done successfully by partitioning the required density into a pair of components in conformity with the formula $w(t) = \varphi^*(t) + \overline{\chi^*(t)}$, where $\varphi^*(z)$ and $\chi^*(z)$ are some regular functions in a circle bounded by L_2. For such a separation of the density, all the integrals mentioned are taken in closed form in terms of the composite functions $\varphi^*(z)$ and $\chi^*(z)$ (and their conjugates) of the arguments dependent on a small parameter, for which it is natural to take the quantity $\varepsilon = 1 - R_1 / R_2$. Having computed all the integrals with density $w(t)$ in (1.1) by such means, let us replace the latter by a functional relationship; it is separated into two relations including subdivided functions regular inside and outside L_2, respectively. In each of these relationships such functions can be expanded in series of the small parameter. It turns out unexpectedly that a direct comparison of these expansions results, in turn, in an expansion with initial terms on the order of $O(\varepsilon^3)$. This suggests that the $\Phi(z)$ and $\Psi(z)$ sought will grow in absolute value as $O(\varepsilon^{-3})$ as the ring becomes thinner.

A qualitatively analogous phenomenon (with the confidence of the conclusion made) should evidently be observed even in an analysis of the solution, obtained by another method, for a ring by means of Fourier series, say. The similar result hence extracted implies the correctness of the more general method of investigation used earlier, which is based on using the Fredholm integral equation.

Note. A closed circular cylindrical shell (of finite length) with free transverse edges was considered in [4]. Starting from two-dimensional equations of the general theory, the authors showed that the deflection w is on the order of ε^{-3} in this case, i.e. is the same as that obtained here for the displacement vector component (the deflection is $w = O(\varepsilon^{-1})$ for sufficiently stiffly clamped shell edges).

2. Furthermore, following Muskhelishvili, let us use the form of the solution of the problem for a concentric ring in polar coordinates in terms of complex function theory. The desired Kolosov-Muskhelishvili functions are taken in the form of Laurent series with coefficients to be determined [5]

$$\Phi(z) = \sum_{k=-\infty}^{\infty} a_k z^k, \quad \Psi(z) = \sum_{k=-\infty}^{\infty} a_k' z^k$$

where for the external forces given by a general complex Fourier series, the coefficients a_k $(k = \pm 2, \pm 3, \ldots)$ are given by the explicit expressions

$$a_n = \frac{(1+n)(R_2^2 - R_1^2)\, B_n - (R_2^{-2n+2} - R_1^{-2n+2})\, \bar{B}_n}{(1-n^2)(R_2^2 - R_1^2)^2 - (R_2^{2n-2} - R_1^{2n+2})(R_2^{-2n+2} - R_1^{-2n+2})}$$

The denominator of this fraction is reduced after some transformations to

$$R^4 (1-\lambda)^2 [(1-n^2) + P_n(\lambda)] = R^4 (1-\lambda)^4 q_n(\lambda) \tag{2.2}$$

$$P_n(\lambda) = \frac{(1-\lambda^{n+1})(1-\lambda^{n-1})}{\lambda^{n-1}(1-\lambda)^2}, \quad q_n(\lambda) = \sum_{k=1}^{n-1} (n-k)\lambda^{-k} \left(\frac{1-\lambda^k}{1-\lambda}\right)^2$$

$$\lambda = (R_1 / R_2)^2$$

Let us briefly clarify the derivation of (2.2). We have directly

$$P_n(\lambda)=\frac{1}{\lambda^{n-1}}\sum_{l=0}^{n}\mu_l\lambda^l\sum_{k=0}^{n-2}m_k\lambda^k,\quad \mu_l=m_k=1$$

or, having set $e+k=k_1$ and omitting the one subscript on the k_1, after replacement of the second index of summation, we obtain

$$P_n(\lambda)=\frac{1}{\lambda^{n-1}}\sum_{l=0}^{n}\mu_l\sum_{k=l}^{n+l-2}m_{k-l}\lambda^k$$

As is seen, the double summation on the right is taken over integer points of the closed parallelogram $OABC$ (Fig. 1). Upon a mutual displacement of the aggregates forming

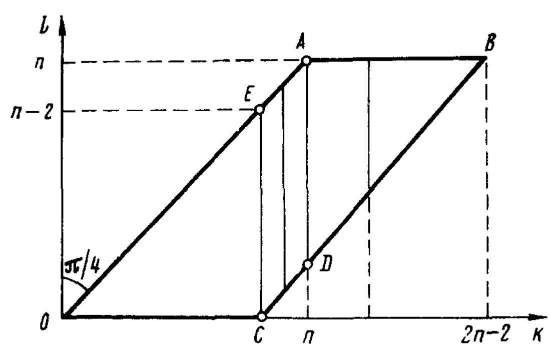

Fig. 1

this double series, it is necessary to perform the summation in totality of integer points of the triangle OEC, of the parallelogram $ECDA$, and then the triangle ABD only taking integer points of the common boundaries of adjacent regions once. Doing this successively, we find

$$P_n(\lambda)-\frac{1}{\lambda^{n-1}}\left\{\sum_{k=0}^{n-2}\lambda^k\sum_{l=0}^{k}\mu_l m_{k-l}+\sum_{k=n-1}^{n-1}\lambda^k\sum_{l=1}^{n-1}\mu_l m_{k-l}+\sum_{k=n}^{2n-2}\lambda^k\sum_{l=k-(n-2)}^{n}\mu_l m_{k-l}\right\}$$

This last equality reduces to a simpler form (since $\mu_l=m_l=1$)

$$P_n(\lambda)=\frac{1}{\lambda^{n-1}}\left[\sum_{k=0}^{n-2}(k+1)\lambda^k+(n-1)\lambda^{n-1}+\sum_{k=n}^{2n-2}(2n-k-1)\lambda^k\right]$$

Furthermore, we convert the first of the sums on the right to the index of summation $k_1=-(k-n+1)$ which yields

$$\frac{1}{\lambda^{n-1}}\sum_{k=0}^{n-2}(k+1)\lambda^k=\sum_{k=1}^{n-1}(n-k)\lambda^{-k}$$

Let us perform the replacement $k_1=k-n+1$ in the second sum, then

$$\frac{1}{\lambda^{n-1}}\sum_{k=n}^{2n-2}(2n-k-1)\lambda^k=\sum_{k=1}^{n-1}(n-k)\lambda^k,\quad \sum_{k=1}^{n-1}(n-k)=\frac{(n-1)n}{2}$$

We now have

$$(1-n^2)+P_n(\lambda)=(1-\lambda)^2\sum_{k=1}^{n-1}(n-k)\lambda^{-k}\left(\frac{1-\lambda^k}{1-\lambda}\right)^2$$

which agrees with (2. 2).

Under the assumption that the normal and tangential stresses are given on the outer and inner circles L_2 and L_1 by the conditions

$$N-iT=e^{in\vartheta}\quad\text{on } L_2,\quad N-iT=0\quad\text{on } L_1$$

where n is a fixed positive integer, the required functions appear as

$$\Phi(z)=\frac{1}{(1-\lambda)^3q_n(\lambda)}\left[(1+n)\left(\frac{z}{R_2}\right)^n-\frac{1-\lambda^{n+1}}{1-\lambda}\left(\frac{R_2}{z}\right)^n\right]$$

$$\Psi(z)=\frac{1}{1-\lambda^{n-1}}\left(\frac{z}{R_2}\right)^{n-2}+\frac{1}{(1-\lambda)^3q_n(\lambda)}\left[(1-n)^2\frac{1-\lambda^n}{1-\lambda^{n-1}}\left(\frac{z}{R_2}\right)^{n-2}-\right.$$
$$\left.(1+n)\lambda\frac{1-\lambda^n}{1-\lambda}\left(\frac{R_2}{z}\right)^{n+2}\right]$$

Performing all the computations successively (omitted here for brevity), we obtain

$$\Phi(z)=\frac{3}{2n^2(n-1)\varepsilon^3}\left\{\left[1-\frac{1+2\delta n}{2}\varepsilon\right]e^{in\vartheta}-\left[1-\frac{1+2(1-\delta)n}{2}\varepsilon\right]e^{-in\vartheta}\right\}$$

$$\bar{z}\Phi'(z)+\Psi(z)=\frac{3}{2(n-1)n\varepsilon^3}\left[\Lambda_1(\delta,\varepsilon)e^{i(n-2)\vartheta}+\Lambda_2(\delta,\varepsilon)e^{-i(n-2)\vartheta}\right]$$

$$z=Re^{i\vartheta}\quad(R_1\leqslant R<R_2)$$

$$\Lambda_1(\delta,\varepsilon)=(1-2\delta)\varepsilon+[(2n-3)\delta^2-(n-3)\delta-1/3]\varepsilon^2+\ldots$$

$$\Lambda_2(\delta,\varepsilon)=(1-2\delta)\varepsilon-\left[(2n+3)\delta^2-3(n+1)\delta+\frac{2n+1}{3}\right]\varepsilon^2+\ldots,$$

$$\delta=\frac{R_2-R}{R_2-R_1}$$

We hence arrive at the formulas for the principal values of the stress components

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\end{Bmatrix}=\frac{3(1-2\delta)}{2n(n-1)\varepsilon^2}\{2\cos n\vartheta\mp[\cos(n-2)\vartheta+\cos(n+2)\vartheta]\}\qquad(2.3)$$

$$\tau_{xy}=\frac{3(1-2\delta)}{2n(n-1)\varepsilon^2}[\sin(n-2)\vartheta-\sin(n+2)\vartheta]$$

As regards the components of the displacement vector, they retain the order of the Goursat function, equal to $O(\varepsilon^{-3})$ (because no mutual cancellation of the components with the order $O(\varepsilon^{-3})$ generally occurs in the presence of a constant $\varkappa$ dependent on the elastic properties of the medium in their expressions; here a direct analogy with the results in [1] is observed).

Note. Let us establish a curious fact. Let us supplement the principal part (extracted by the manner mentioned) of the magnitude of any of the stress components with a quantity corresponding to the additional component in the external load, which equals $A_m e^{im\vartheta}(m\neq n)$. Then in the sum comprised of the main component corresponding to the external force determined by the quantity $e^{in\vartheta}$ in the boundary condition, and the additional component mentioned, the stress component taken can be made to vanish at $\vartheta=\vartheta_0$ by defining the constant A_m in the manner needed (ϑ_0 is the value of the polar angle selected at random). It can happen that the magnitude of the component introduced supplementarily takes the value zero; in other words, the added expression (considered by itself) itself possesses the required property and, consequently, is sought.

Now, let the external forces applied to the outer boundary L_2 be given by a polynomial in the form of a finite segment of the Fourier series

$$N - iT = A_0 e^{in_0\vartheta} + \sum_{\nu=1}^{3q} A_\nu e^{in_\nu\vartheta} \tag{2.4}$$

The A_ν here are real quantities (they can be selected from any kind of reasoning), n_ν are some positive integers (in general, also fixed). We hence have for the principal parts of the stress components

$$\sigma_x \approx \frac{1-2\delta}{\varepsilon^2}\left[A_0 h_0(n_0, \vartheta) + \sum_{\nu=1}^{3q} A_\nu h_\nu(n_\nu, \vartheta)\right]$$

$$\sigma_y \approx \frac{1-2\delta}{\varepsilon^2}\left[A_0 l_0(n_0, \vartheta) + \sum_{\nu=1}^{3q} A_\nu l_\nu(n_\nu, \vartheta)\right]$$

$$\tau_{xy} = \frac{1-2\delta}{\varepsilon^2}\left[A_0 \omega_0(n_0, \vartheta) + \sum_{\nu=1}^{3q} A_\nu \omega_\nu(n_\nu, \vartheta)\right]$$

In order to extract the first component, let us take $A_0 = 1$, $n_0 = n$ and $h_0(n_0, \vartheta) = h(n, \vartheta)$, $l_0(n_0, \vartheta) = l(n, \vartheta)$, $\omega_0(n_0, \vartheta) = \omega(n, \vartheta)$, respectively. Furthermore, let us require that the principal parts of the stress components vanish simultaneously at the radial sections $\vartheta = \vartheta_\nu$ $(\nu = 1, \ldots, q)$. We hence arrive at the conditions

$$h(n, \vartheta_\nu) + \sum_{\mu=1}^{3q} A_\mu h_\mu(n_\mu, \vartheta_\nu) = 0 \tag{2.5}$$

$$l(n, \vartheta_\nu) + \sum_{\mu=1}^{3q} A_\mu l_\mu(n_\mu, \vartheta_\nu) = 0 \quad (\nu = 1, 2, \ldots, q)$$

$$\omega(n, \vartheta_\nu) + \sum_{\mu=1}^{3q} A_\mu \omega_\mu(n_\mu, \vartheta_\nu) = 0$$

The relationships obtained form a system of algebraic equations to determine the constants A_μ $(\mu = 1, 2, \ldots, 3q)$, which we find under the assumption that the system determinant is nonzero. When this determinant takes on a zero value, it is necessary to set $A_0 = 0$ from the very beginning and then we obtain a homogeneous system of $3q$ equations of the same structure (and with the same determinant) in place of (2.5). We hence find the required quantities A_μ (not all of which are zero) which assure conservation of the relationships

$$\sigma_x = \sigma_y = \tau_{xy} = 0|_{\vartheta=\vartheta_\nu} \quad (\nu = 1, 2, \ldots, q)$$

It is clear from the above that the magnitudes of stress components on the order of $O(\varepsilon^{-1})$ acquire a leading role for $\vartheta = \vartheta_\nu$ for a load of type (2.4) with coefficients A_μ to be determined (as mentioned above). Evidently, by following the same considerations we easily see that all three stress components differ not only by the reduced growth index (for decreasing ε) but generally remain bounded in q arbitrarily selected radial sections with the polar angles $\vartheta_1, \vartheta_2, \ldots \vartheta_q$.

The author is grateful to A. L. Gol'denveizer for discussing the research and for useful remarks.

REFERENCES

1. Ustinov, Iu. A., On the analysis of belted rings. Izv. Akad. Nauk SSSR, Mekhan. i Mashinostr., № 3, 1964.
2. Ustinov, Iu. A., Stress concentration in a half-plane and a plane with circular holes under tension. Izv. Akad. Nauk SSSR, Mekhanika, № 1, 1965.
3. Sherman, D. I., On stresses in a heavy half-plane weakened by two circular openings. PMM Vol. 15, № 3, 1951.
4. Gol'denveizer, A. L. and Zveriaev, E. M., State of stress of unconstrained shells of zero curvature. PMM Vol. 35, № 2, 1971.
5. Muskhelishvili, N. I., Some Basic Problems of the Mathematical Theory of Elasticity. (English translation), Groningen, Noordhoff, 1953.

Translated by M. D. F.

UDC 539.3: 534.1

DIFFRACTION OF ACOUSTIC WAVES IN A PLANE SEMI-INFINITE WAVEGUIDE WITH ELASTIC WALLS

PMM Vol. 40, № 1, 1976, pp. 104-111

D. P. KOUZOV and V. A. PACHIN
(Leningrad)

(Received July 31, 1974)

Exact analytic expression for Green's function of the Helmholtz equation for the half-strip and boundary conditions that contain high order derivatives is obtained by the method of expansion in terms of plane waves. This problem arises in the determination of the acoustic field created by a point source in a plane semi-infinite acoustic waveguide with thin elastic walls, and also inside an infinite acoustic waveguide with a thin elastic baffle.

1. Statement of the problem. Examples. We seek the solution of the problem

$$(\Delta + k^2) P(x, y) = -\delta(x - x_0, y - y_0), \quad 0 < x < \infty, \; 0 < y < h \tag{1.1}$$

$$L_\alpha P(x, y_\alpha) = 0, \quad 0 < x < \infty \quad \alpha = 1, 2; \quad y_1 = 0, \quad y_2 = h \tag{1.2}$$

$$L_3 P(0, y) = 0, \quad 0 < y < h \tag{1.3}$$

$$L_\alpha = (-1)^{\alpha+1} m_{\alpha 1}\left(-\frac{\partial^2}{\partial x^2}\right)\frac{\partial}{\partial y} + m_{\alpha 2}\left(-\frac{\partial^2}{\partial x^2}\right), \quad \alpha = 1, 2$$

$$L_3 = m_{31}\left(-\frac{\partial^2}{\partial y^2}\right)\frac{\partial}{\partial x} + m_{32}\left(-\frac{\partial^2}{\partial y^2}\right)$$

where P is the acoustic pressure in the medium, Δ is the Laplace operator, k is the wave number, the time dependence is specified by the factor $e^{-i\omega t}$ which is omitted throughout, $m_{\alpha\beta}$ are polynomials of their arguments whose coefficients are independent of space coordinates x and y. In the considered region the sought solution must be continuous up to the boundary, with the exception of point (x_0, y_0) of location of the source, and must satisfy the principle of ultimate absorption.

For the simplest Dirichlet or Neumann boundary conditions the considered problem